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Hot-Wire Probes for Measuring Velocity and Concentration in Helium-Air Mixtures

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Introduction

There is a paucity of reliable data on fluctuating quantities in turbulent flows with variable density, principally because of the difficulties connected with making the appropriate measurements. One means of providing such data involves measuring with time-resolution the velocity and concentration of a foreign gas, e.g., of helium, in a low-speed isothermal flow with air as the second component. The mixing of ambient-temperature helium discharging at low speed from a circular orifice into quiescent air is an example of such a flow. By this means and by use of tape-recording techniques, the experimental difficulties, although formidable, would appear to be reduced and to be focused on the technique for sensing velocity and concentration.

One such technique, which we have been studying, is based on the suggestion of Corrsin[1] and involves two or more hot wires of sufficiently different characteristics so that from their instantaneous voltage signals one or more velocity components and the concentration of foreign gas can be inferred. Our purpose here is to report on the application of this technique to helium-air mixtures and in particular on a mode of hot-wire operation that appears to provide more promise than a conventional mode. We are interested in helium as the foreign gas because it makes feasible, with equipment available to us, the study of turbulent flows with significant density fluctuations.

We make several introductory remarks. To our knowledge, there have been three recent attempts to apply hot-wire anemometry to the measurement of velocity and concentration in turbulent flows of helium-air mixtures (cf., e.g., Tombach[2]). None have been very successful.‡ We describe our approach and results in terms of two wires used to determine one velocity component, say u, and the mass fraction of helium, denoted by c. Extension to a three-wire probe for measuring u, v, and c appears to be possible. Finally, we consider constant temperature operation with one sensor a wire and the second a film on a quartz fiber. Accordingly, we use the subscripts w and f to identify the two sensors.

Conventional Operation

The basic notion of the hot-wire technique for the present application is that the usual calibration based on King's law, i.e., on a voltage squared, $(u)^{1/2}$ relationship, must be extended to concentration so that the "calibration constants" become calibration functions of concentration. According to this notion, our two sensors follow the relations

$$E_w^2 = A_w(c) + B_w(c)(u)^{1/2}$$
$$E_f^2 = A_f(c) + B_f(c)(u)^{1/2} \tag{1}$$

With the functions $A_w(c)$, $B_w(c)$, $A_f(c)$, and $B_f(c)$ known from calibration plus some curve-fitting of discrete data in c, then a pair of voltages, obtained during data collection, yield implicitly the concentration according to the following relation derivable from Eqs. (1):

$$E_w^2 = A_w[1 - (B_w/B_f)(A_f/A_w)] + (B_w/B_f)E_f^2 = a(c) + b(c)E_f^2 \tag{2}$$

With c determined from Eq. (2) either of Eqs. (1) yields u.

Now from King's law and the dependence of the fluid properties appearing therein on c, one can make estimates of the calibration functions for given sensors operating at given temperatures. Such estimates show that $a(c)$ is proportional to the thermal conductivity of the mixture and is thus a sensitive function of concentration. The calibration function b is essentially independent of concentration. However, we know from earlier work (cf. Refs. 3 and 4) that thermal slip effects due to the poor thermal accommodation of helium on most hot-wire materials makes suspect the application of King's law in helium-air mixtures. Thus experiment must be resorted to in order to assess the applicability of Eqs. (1) and (2) for present purposes.

In addition to establishing practical data on heat loss, sensitivity, etc. a certain amount of experimentation appears to be required in our probe development in order to obtain spatial resolution without interference between sensors; to achieve stable, reproducible performance from the sensors, etc. We report here on the results of a series of tests carried out on a wire-film combination consisting of a platinum wire of 0.0001 in. diam and 0.015 in. length and of a film of 0.001 in. diam with an active length of platinum of 0.010 in. The wire is unswept and is mounted in a plane orthogonal to the axis of the film at its center. The parameters that distinguish

Received January 9, 1970. This work was sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, U.S. Air Force, under Grant AF-AFOSR-927A-67. The results reported here relate to a project of larger scope, in the course of which the authors have benefited greatly from discussions with, and suggestions from, many people. We wish to explicitly acknowledge the help of J. LaRue in collecting the data presented here.

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‡ We have not experienced the "history" effect that is reported by Tombach and that caused him to abandon the hot-wire technique. However, in our mode of calibration and data collection, there is no occasion for us to expose the probe to high concentrations for long periods of time.

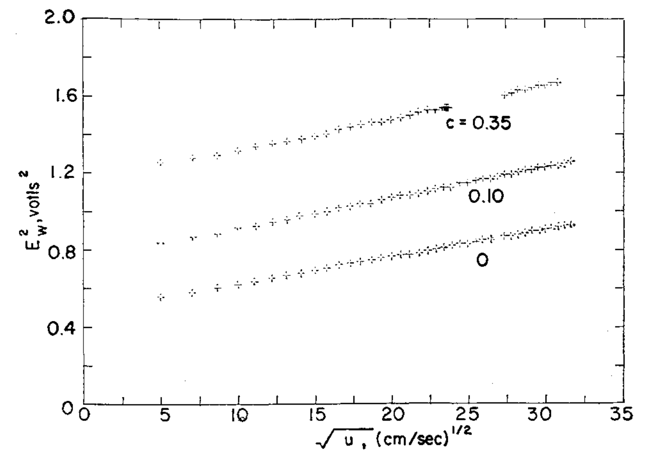

Fig. 1 Heat loss characteristics for wire upstream 0.050 in. with ΔT = 50°C.

the results are the wire and the film temperatures, the separation distance between the two sensors, and whether the wire or the film is upstream of the other. We note that to avoid significant interference the upstream sensor must be at a low overheat and large separation.

We discuss first results obtained when the sensors are operated so that the interference between them is insignificant and so that Eqs. (1) and (2) might be expected to pertain. Consider the case when the wire is 0.050 in. upstream of the film and is operated at low overheat, 50°C above ambient, while the film is at 275°C above ambient. That Eqs. (1) apply is shown in Figs. 1 and 2 wherein calibration curves for air (c = 0), and for c = 0.10, 0.35 are shown. However, Fig. 3 displays graphically Eq. (2) and shows that both $a(c)$ and $b(c)$ therein are relatively weak functions of concentration, so weak as to make determination of c from Eq. (2) rather inaccurate. Thus the dependence of $a(c)$ on thermal conductivity appears to be greatly weakened by thermal slip effects. We remark that there appears to be no theory adequate to predict these slip effects in the Reynolds number range of interest here.

A similar negative result is obtained when the film is operated at low overheat, 50°C above ambient, 0.050 in. upstream of the wire with 500°C overheat. Again in this case the individual sensors follow closely Eqs. (1) but when combined in the form of Eq. (2) the results are as shown in Fig. 4.

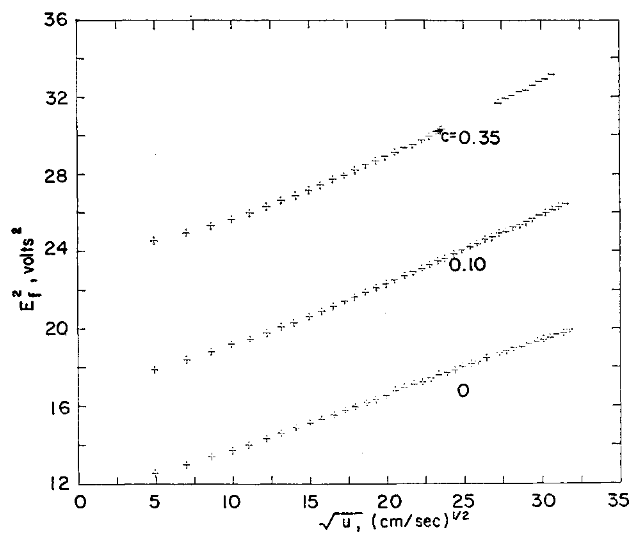

Fig. 2 Heat loss characteristics for film downstream 0.050 in. with ΔT = 275°C.

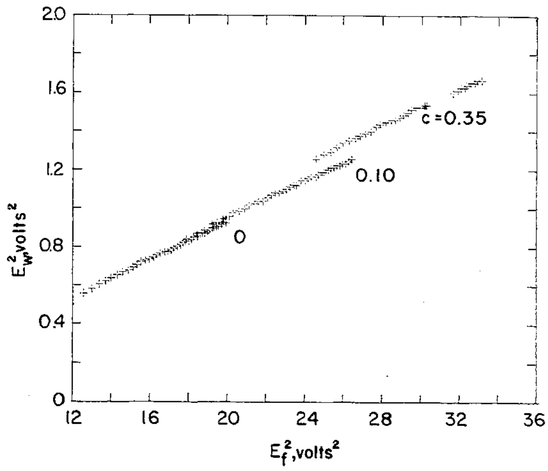

Fig. 3 Combined heat loss characteristics from Figs. 1 and 2.

The results shown on Figs. 1–4 are typical of those we have obtained from a variety of two- and three-sensor probes operated in various configurations and at various temperatures. From this experience we have concluded that over the limited velocity range of interest in the usual application of a given probe, Eqs. (1) apply but that it is difficult, presumably because of thermal slip effects, to achieve with noninterfering sensors sufficient sensitivity to concentration so that Eqs. (2) provide a useful means for determining c. In other words, it is difficult to distinguish voltage pairs corresponding to high velocity-low concentration from those corresponding to a lower velocity-higher concentration.

An Interfering Probe

We find that the sensitivity to concentration can be greatly enhanced by allowing the thermal fields of the film and wire to interfere. Consider, for example, a configuration such as that leading to the results shown in Figs. 1–3 but with a small separation so that the wire is in the thermal field of the film. Thus we are clearly abandoning the first of Eqs. (1). At low velocities and high concentrations of helium some of the heat loss from the wire is made up by heating from the film. In fact, at low enough velocities and high enough concentrations no electrical input would be required to maintain the wire at its fixed temperature and no signal is obtained. This is undesirable, of course, and the separation and wire temperature must be sufficiently large so this does not occur in the desired operating range of the probe. On the other hand at high velocities and low concentrations of helium the thermal field of the film is closely confined to

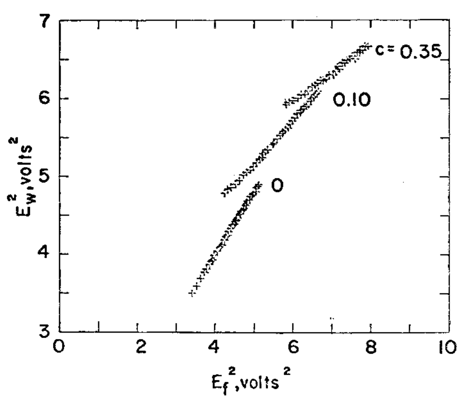

Fig. 4 Combined heat loss characteristics for wire downstream 0.050 in. at ΔT = 500°C, film at ΔT = 50°C.

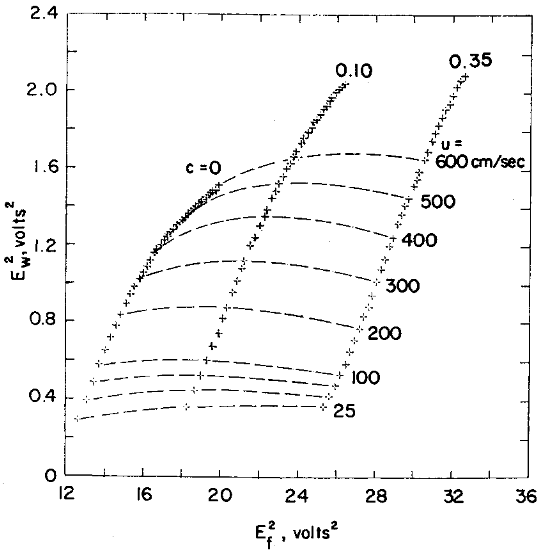

Fig. 5. Combined heat loss characteristics for wire upstream 0.002 in. at $\Delta T = 100°C$, film at $\Delta T = 275°C$.

the film, the wire is uninfluenced by the film, and the noninterfering behavior shown in Figs. 1–3 is recovered. Again this is undesirable.

To indicate the sort of sensitivity that can be obtained by the configuration under discussion we show in Fig. 5 the performance of a probe with the upstream wire separated by 0.002 in. from the film and operated at 100°C overheat. The dramatic increase in sensitivity to concentration is apparent from a comparison of Figs. 3 and 5. We add to the lines of constant concentration in Fig. 5 isovelocity lines in order to indicate the concentration-velocity range for which there can be realized superior sensitivity. We find from Fig. 5 that for a velocity range with a lower end of $u = 25$ cm/sec and an upper end depending on concentration, $E_w^2 \simeq \bar{a}(c) + \bar{b}E_f^2$ where $\bar{b}$ is constant so that a simple means for obtaining c is evident. In addition the film is found to be unaffected by the wire so that Fig. 2 still applies. Thus we can readily invert from voltage pairs to u and c pairs.

Concluding Remarks

In conclusion we note that a probe similar to that resulting in the data of Fig. 5 but with greater sensitivity to concentration at high velocities and low concentrations can be obtained by increasing the film temperature and that a relatively cool "X-wire" in the thermal field of the film would appear to provide a means for measuring u, v, and c. We also note that other interfering configurations can be considered, e.g., a relatively cool wire downstream of a film, but the configuration leading to the results of Fig. 5, although perhaps not unique, does have the great virtue of having nearly orthogonal contours of constant concentration and constant velocity. Other interfering sensors we have tried lead to corresponding contours which are highly skewed and thus to degraded accuracy in separating c and u.

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Effect of Spin on the Velocity of a Re-Entry Body

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1. Introduction

THE purpose of the present Note is to derive an approximate expression for the velocity of a rolling re-entry body as a function of roll, pitch, and yaw rates as well as the re-entry angle of attack. The present analysis is an extension of the analysis of Ref. 1.

In the analysis, the rotational motion is decoupled from the translational motion. A closed form expression for the angle of attack is given. This expression for the angle of attack is then used to calculate the drag of the body, which is used in turn to determine the variation of the velocity with altitude. The results of the analysis are in good agreement with six-degree-of-freedom numerical computations.

2. Angle of Attack

A number of investigators[2–7] determined the angle of attack allowing for variable velocity, dynamic pressure, and aerodynamic derivatives. References 6 and 7 allowed also for small mass and aerodynamic asymmetries as well as roll accelerations. For a constant roll rate, and for an axisymmetric body, the complex angle of attack $\delta = \beta + i\alpha$ can be written as[6]

$$\delta = [R_1/(V\omega)^{1/2}]e^{-\Lambda_1}e^{i\Omega_1} + [R_2/(V\omega)^{1/2}]e^{-\Lambda_2}e^{i\Omega_2} \tag{1}$$

$$\Omega_{1,2} = \int_0^t \left(\frac{\omega \pm pI_x}{2I}\right) dt \tag{2}$$

$$\Lambda_{1,2} = \int_0^t (\lambda \pm \Delta\lambda)dt \tag{3}$$

$$\omega = -[(C_{m\alpha}AdV^2/2I)\rho + (I_xp/2I)^2]^{1/2}$$

$$\lambda = -(C_{N\alpha}/m - C_{mq}d^2/2I)(AV/4)\rho$$

$$\Delta\lambda = -(C_{N\alpha}/m + C_{mq}d^2/2I)AVI_xp/8\omega I\ \rho$$

where m is the body mass, A area, d diameter, I_x roll moment of inertia, I transverse moment of inertia, V velocity, $C_{N\alpha}$ normal force derivative, $C_{m\alpha}$ pitching moment coefficient, and C_{mq} is pitching or yawing damping coefficient. In this solution, the angle of attack is assumed small, but the velocity, dynamic pressure, and aerodynamic derivatives are assumed to be slowly varying in comparison with the angle-of-attack oscillations. The complex constants of integration R_1 and R_2 are determined from the re-entry initial conditions (denoted by e); that is

$$R_1 = (\delta_e - I\zeta_e/I_xp)(V_e\omega_e)^{1/2} \tag{4}$$

$$R_2 = (I\zeta_e/I_xp)(V_e\omega_e)^{1/2} \tag{5}$$

where $\zeta_e = q_e + ir_e$ with q_e and r_e the initial pitch and yaw rates.

For a high-performance re-entry vehicle, the velocity and trajectory path angle change appreciably only at the lower altitudes where the angles of attack have converged. Thus, the velocity and path angle in Eqs. (1–3) will be assumed constant. In addition, the aerodynamic derivatives are assumed constant. Assuming an exponential atmosphere;

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